(1) Let $p =$ probability for success $= 0.92$.

$=$ pop. proportion of seeds from samples of size $n = 160$ that actually germinate.

We want to use the sampling distribution model for $\hat{p}$ to calculate $P(\hat{p} > 0.95)$.

Conditions Check

(1) Bernoulli Trials are independant. Whether or not any one seed germinates does not depend on whether or not any other seed germinates.

(2) Randomization. We assume any package of 160 seeds is a random sample of seeds

(3) n is less than 10% of the pop. $n = 160$ is much less than 10% of the pop of all seeds.

(4) $np \geq 10$ and $nq \geq 10$

$n \cdot p = 160 \cdot (0.92) \doteq 147$

$n \cdot q = 160\,(0.08) \doteq 13$

The conditions all check, so I can use a normal model to approximate the sampling distribution of all possible sample proportions, $\hat{p}$ from samples of size $n = 160$

Then, $P(\hat{p} > 0.95) = P\left(z > \dfrac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}\right)$

$$= P\left(z > \frac{0.95 - 0.92}{\sqrt{\frac{(.92)(0.08)}{160}}}\right) \simeq P(z > 1.40)$$

$= \text{normalcdf}(1.40, 10)$

$\approx \boxed{0.0808}$

(2) $n = 150$

Suppose $p = 0.08$ = the population proportion of rotten apples on the truck.

Use the sampling distribution model for sample proportions.

A sample of 150 apples taken from a truckload of apples with 8% defectives may not actually reflect 8% defectives.

We will try to use the sampling distr. model to calculate

$P(\hat{p} \leq 0.05)$, since the shipment will be accepted if less than or equal to 5% of apples in the sample are rotten.

Conditions Check
1. Trials are independent ✓
2. Sample is a random Sample ✓
3. $n = 150$ is less than 10% of the pop of all apples on the truck. ✓
4. $np = 150 \cdot (.92) = 138$ and $n \cdot q = 12$

   so $np \geq 10$ and $nq \geq 10$ ✓

$$P(\hat{p} \leq 0.05) = P\left(z \leq \frac{0.05 - 0.08}{\sqrt{\frac{(.08)(0.92)}{150}}}\right) \approx P(z \leq -1.35)$$

$= \text{normalcdf}(-10, -1.35)$

≈ 0.0885

(3) Binomial Experiment with $n = 865$ voters and

$X =$ the number of successes (a voter approves of the issue)

in n trials.

Given $\hat{p} = \frac{X}{n} = \frac{408}{865}$, find a 90% conf. interval estimate

for p, the population proportion of voters who favor the issue.

Conditions Check

1. Independent trials ✓
2. Random sample ✓
3. $n = 865$ is less than the pop. of all voters
4. $n\hat{p} \geq 10$ and $n\hat{q} \geq 10$

$n \cdot \hat{p} = 408$ and $n \cdot \hat{q} = 457$

$$ME = z^* \cdot SE(\hat{p})$$

$$= 1.645 \cdot \sqrt{\frac{\hat{p}\,\hat{q}}{n}}$$

$$= 1.645 \sqrt{\frac{\frac{408}{865} \cdot \frac{457}{865}}{865}}$$

$$\approx 0.027920933$$

Then, an interval estimate of p is $\hat{p} - ME < p < \hat{p} + ME$, or

$$\frac{408}{865} - ME < p < \frac{408}{865} + ME, \text{ or}$$

$$0.44 < p < 0.50.$$

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(36) Sample size determination formula for a pop. proportion.

Use $n = \frac{(z^*)^2 \hat{p} \cdot \hat{q}}{(ME)^2}$ if $\hat{p}$ & $\hat{q}$ are known / given, or use $n = \frac{(z^*)^2 \cdot 0.5}{(ME)^2}$ otherwise.

For this problem, we want to try to get $\frac{1}{2}$ of the ME from before, to get a better estimate. So, we use the first formula, but we replace ME with $\frac{1}{2}$ of the ME calculated above. $n = \frac{(1.645)^2 \frac{408}{865} \cdot \frac{457}{865}}{\left(\frac{1}{2} \cdot 0.0297920933\right)^2} = \boxed{3{,}460 \text{ voters}}$

(4) When you draw this sample, it is a Binomial Experiment with $n = 2025$ and success defined as a test subject experiences significant relief from the allergy drug. Then,

X = the discrete random variable representing the number of successes in n trials; and $\hat{p} = \frac{X}{n} = \frac{1742}{2025}$

a) Assume the new treatment works with the same (or worse) level of effectiveness as the number one brand; i.e., that the population proportion of people experiencing relief is 83% (or less).

H_0: $p = 0.83$ (or $p \leq 0.83$)

H_a: $p > 0.83$

We want to sample the population and determine if our sample proportion value (which will be different from the hypothesized value 83%) is due to sampling variability; or if it's the case that our assumption that the null hypothesis is true is, in fact, really false.

Since the conditions (independence, randomization, < 10% of the pop, $np \geq 10$ and $nq \geq 10$) check, We use the sampling distribution model to answer the question:

What is the probability that we can get a sample proportion greater than or equal to the one we got, given we assume $p = 0.83$?

In other words, what is $P\left(\hat{p} > \frac{1742}{2025}\right)$?

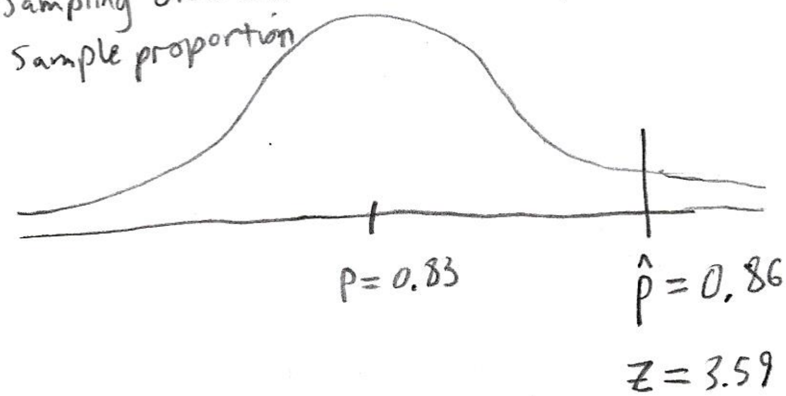
$H_a: p > 0.83$

b) The Z-score associated with $\hat{p} = \frac{1742}{2025} \doteq .86$

is $Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.86 - 0.83}{\sqrt{\frac{(.83)(.17)}{2025}}} = \frac{0.03}{0.008347396}$

≈ 3.59

Sampling distribution model for the sample proportion



c) Then, the p-value is

$P(\hat{p} > 0.86) = P(Z > 3.59) = \text{normalcdf}(3.59, 10)$

(5a) Conditions Check

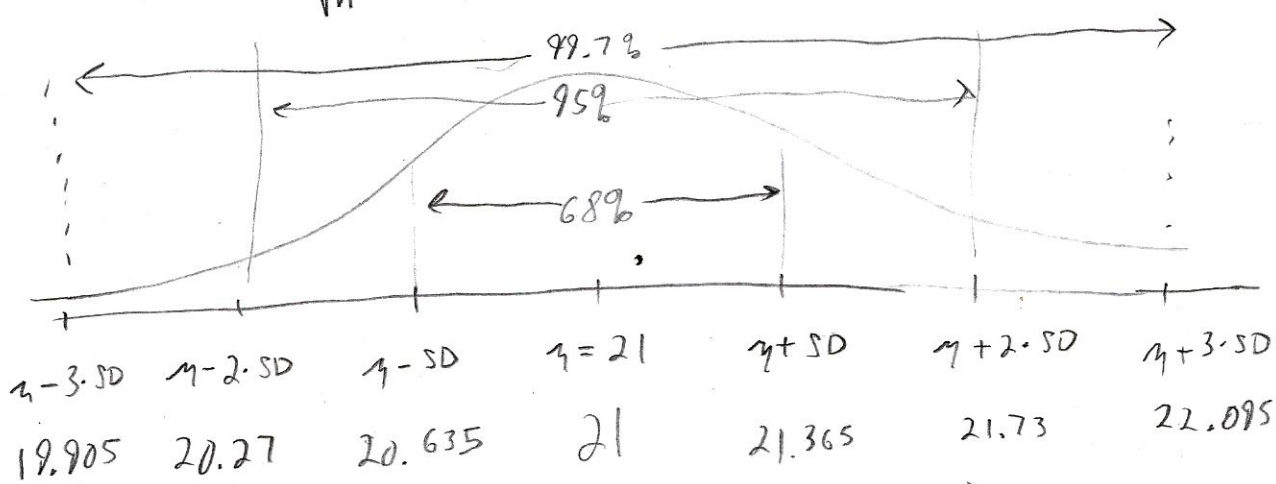
✓ Randomization: we are told the sample is random.

✓ 10% Condition: 30 pieces of paper is less than all of the pieces of paper manufactured in an hour.

✓ Large enough sample: 30 pieces is large enough.

By the central limit theorem,
The sampling distribution of sample means ($\bar{x}$) is normal with $\mu = 21$

and $SD(\bar{x}) = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{30}} \approx 0.365$.

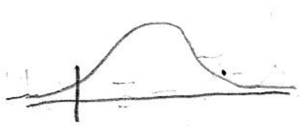


(5b) Find $P(\bar{x} < 20) = P\left(z < \frac{\bar{x} - \mu}{SD(\bar{x})}\right) \doteq P\left(z < \frac{20 - 21}{.365}\right)$

$\approx P(z < -2.74) = \text{normalcdf}(-10, -2.74)$

$\doteq 0.003072$

(5c)



We want the percentage of sample means left of $\bar{x}$ to be 1/1000, or 0.001. We solve the z-score formula, $z = \frac{\bar{x} - \mu}{SD(\bar{x})}$ for μ with $\bar{x} = 20$,

(6) Conditions Check:

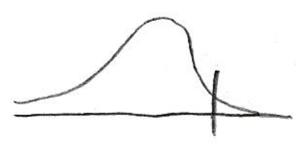
✓ Randomization: we assume the sample will be a random sample

✓ 10% condition: $n=25$ washing machines seems like it's less than the number of All washing machines of this model.

✓ Large Enough Sample: we are told the population of all replacement times for this model of washing machine is normally distributed. So, the CLT applies to means of small samples; 25 is sufficent.

The pop. of replacement times is a normal model with $\mu=10.3$ and $\sigma=2.7$. The sampling distribution of sample means is also normally distributed, but with $\mu=10.3$ and $SD(\bar{x}) = \frac{\sigma}{\sqrt{n}} = \frac{2.7}{\sqrt{25}} = \frac{2.7}{5} = 0.54$.

The question asks us to use the sampling distribution model to calculate $P(\bar{x} > 12)$. Then,



$\bar{x}=12$

$$P(\bar{x} > 12) = P\left(z > \frac{\bar{x}-\mu}{SD(\bar{x})}\right)$$

$$= P\left(z > \frac{12-10}{0.54}\right) \approx P(z > 3.70)$$

$$= \text{normalcdf}(3.70, 10)$$

$$\approx 1.08 \times 10^{-4}$$ roughly 1 in 10,000.

7a) $H_0: p = 0.50$

$H_a: p \neq 0.50$

7b) $H_0: p = 2/3$

$H_a: p > 2/3$

8) Before managers at the factory draw a random sample, they assume (their null hypothesis) that $p = P_0$, or that the true proportion, p, of defective boxes is some number P_0. They know that whenever the draw a sample, the sample proportion of defective boxes won't be exactly P_0. But, managers use the sampling distribution of sample proportions to find out if the sample proportion ($\hat{p}$) they obtain is unusual or if the difference between P_0 and $\hat{p}$ is just due to sampling variability.

$H_0: p = P_0$

$H_a: p > P_0$

The truth

my decision	Ho true	Ho false
Reject Ho	Type I error	OK
Fail to reject Ho	OK	Type II error

table page 544

a) $\boxed{\text{Type I error}}$ When Ho is really, in fact, true, but we reject it and conclude the alternative is true. In this case, management has concluded the sample proportion they obtained was extreme enough to warrant rejection of Ho, meaning they believe there is a problem with machinery, staff, etc. Production is halted and time (money) is spent trying to fix a non-existant problem.

b) $\boxed{\text{Type II Error}}$ When Ho is really, in fact false but there was a failure to reject Ho. In this case, management has concluded that the proportion of defectives is not greater than P_0, even though their

8b continued

sample proportion, $\hat{p}$, was not equal to p_0.

So, management doesn't think there is a problem, but there really is one. The proportion of defective boxes of Krusty-O's is beyond an acceptable level.

c) Type II

It might cost the company more in the way of lawsuits, expensive recall campaigns (both which damage the Krusty Brand), and damaged equipment repairs, than it does (under the type I scenario) to pay employees to check equipment for damage/failure or double check that the correct procedures were carried out.

d) Type II

⑨ Let

p_1 = population proportion of carpal tunnel sufferers who would show improvement from surgery just after 3 months.

p_2 = pop. proportion of sufferers who would show improvement after 3 months from wearing wrist splints

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We are told that the sample size is $n = 176$.

$$n_1 = \left(\tfrac{1}{2}\right) \text{of} (176) = \tfrac{1}{2} \cdot 176 = 88 \quad \text{and} \quad n_2 = 88$$

We are told $\hat{p}_1 = 0.80$ and $\hat{p}_2 = 0.48$.

ⓐ

$$SE(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}$$

$$= \sqrt{\frac{(0.8)(0.2)}{88} + \frac{(0.48)(0.52)}{88}} \approx 0.068224229$$

ⓑ

$$(\hat{p}_1 - \hat{p}_2) - ME < p_1 - p_2 < (\hat{p}_1 - \hat{p}_2) + ME, \text{ or}$$

$$(\hat{p}_1 - \hat{p}_2) - z^* \cdot SE < p_1 - p_2 < (\hat{p}_1 - \hat{p}_2) + z^* SE, \text{ or}$$

$$(.8 - .48) - 1.96(0.068224229) < p_1 - p_2 < (.8 - .48) + 1.96(0.068224229)$$

$$\boxed{0.25 < p_1 - p_2 < 0.45}$$

ⓒ We are 95% confident that the difference between the two population proportions is between 25% and 45%.

(10) Let P_1 = the population proportion of expecting mothers under age 38

and

P_2 = the pop. prop. of expecting mothers aged 38 or older

We are given

$X_1 = 42$ and $X_2 = 7$

$n_1 = 157$ $n_2 = 89$. So, we conclude that

$\hat{P}_1 = \frac{X_1}{n_1} = \frac{42}{157}$ and $\hat{P}_2 = \frac{X_2}{n_2} = \frac{7}{89}$.

(a)

$H_0: P_1 - P_2 = 0$ (There is no difference in effectiveness)

$H_a: P_1 - P_2 \neq 0$ (There is a difference)

(b) Two-prop-z-test page 566.

$$Z = \frac{\hat{P}_1 - \hat{P}_2}{SE_{pooled}},$$

and $SE_{pooled} = \sqrt{\frac{\hat{P}_{pool}\,\hat{q}_{pool}}{n_1} + \frac{\hat{P}_{pool}\,\hat{q}_{pool}}{n_2}}$

with $\hat{P}_{pool} = \frac{X_1 + X_2}{n_1 + n_2}$ and

$\hat{q}_{pool} = 1 - \hat{P}_{pool}$.

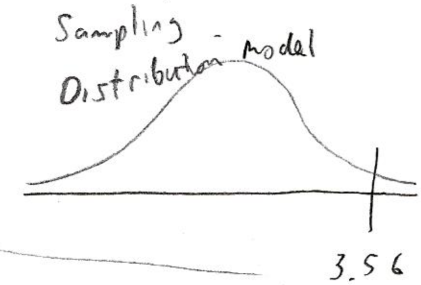
(b) $$\hat{p}_{pool} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{42+7}{157+89} = \frac{49}{246} \doteq 0.199187$$

$$\hat{q}_{pool} = 1 - \hat{p}_{pool} \approx 0.800813$$

$$SE_{pooled} \doteq \sqrt{\frac{(.199187)(.800813)}{157} + \frac{(.199187)(.800813)}{89}}$$

$$\approx 0.052993033$$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{SE_{pooled}} = \frac{\frac{42}{157} - \frac{7}{89}}{0.052993023} \approx 3.56$$



(c) Since we have a 2-tailed test, the p-value is twice the area under the normal curve, right of $z = 3.56$

$$\text{p-value} = 2 \cdot P(z > 3.56) = 2 \cdot \text{normalcdf}(3.56, 10)$$

$$\approx 3.7 \times 10^{-4} = 0.00037$$

37 in 100,000.

Since the p-value is less than 5% or 0.05, we reject Ho and conclude there is a diffeence between the 2 pop. proportions, and, in turn, a diffeence between the efficacy of each treatment.